Galileons, Generalizations & Close Relatives

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Seminar

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Overview

- Some quick motivations
- Galileons - an overview
- Multi-Galileons and Higher Co-Dimension Branes
- Galileons on Curved Spaces - Cosmological Backgrounds
- Comments on ongoing work
- Conclusions.

Acknowledgements & References

- G.Goon, K.Hinterbichler, A.Joyce and M.T., ``*Galileons as Wess-Zumino Terms*," arXiv:1203.3191 [hep-th].
- G.Goon, K.Hinterbichler, A.Joyce and M.T., ``*Gauged Galileons From Branes*," arXiv:1201.0015 [hep-th].
- G.Goon, K.Hinterbichler and M.T., ``*Galileons on Cosmological Backgrounds*," JCAP 1112, 004 (2011) [arXiv:1109.3450 [hep-th]].
- G.Goon, K.Hinterbichler and M.T., ``*A New Class of Effective Field Theories from Embedded Branes*," PRL 106, 231102 (2011) [arXiv:1103.6029 [hep-th]].
- G.Goon, K.Hinterbichler and M.T.,``*Symmetries for Galileons and DBI scalars on curved space*," JCAP 1107, 017 (2011) [arXiv:1103.5745 [hep-th]].
- G.Goon, K.Hinterbichler and M.T., ``*Stability and superluminality of spherical DBI galileon solutions*," PRD 83, 085015 (2011) [arXiv:1008.4580 [hep-th]].
- M.Andrews, K.Hinterbichler, J.Khoury and M.T., ``*Instabilities of Spherical Solutions with Multiple Galileons and SO(N) Symmetry*," PRD 83, 044042 (2011) [arXiv:1008.4128 [hep-th]].
- K.Hinterbichler, M.T. and D.Wesley, ``*Multi-field galileons and higher co-dimension branes*," PRD 82, 124018 (2010)[arXiv:1008.1305 [hep-th]].

Motivations

- Scalar fields appear useful in particle physics and are ubiquitous in cosmology

- Used to break the electroweak symmetry, solve the strong CP problem, inflate the universe, accelerate it at late times, ...

- In most incarnations, the sweet properties of these scalars are offset by their tendency to be most unruly in the face of quantum mechanics.

- Attempts to do away with scalars for some of these tasks, such as modifying gravity, often yield scalars in any case, in limits, or as part of the construction.

- Galileons are an intriguing class of scalars that *may* have a shot at addressing some of these problems, and perhaps most interestingly, are tied to attempts to modify gravity such as massive gravity - you will hear much more about this soon!.

- We'll see - too early to know if these will be useful or not - but it is turning out to be great fun trying.

The Decoupling Limit (of, e.g. DGP)

$$S = \frac{M_5^3}{2r_c}\int d^5x\sqrt{-G}\ R^{(5)} + \frac{M_4^2}{2}\int d^4x\sqrt{-g}\ R$$

Much of interesting phenomenology of DGP captured in the *decoupling limit*:

$$M_4,\ M_5 \to \infty \qquad \Lambda \equiv \frac{M_5^3}{M_4^2} \qquad \text{kept finite}$$

Only a single scalar field - the brane bending mode - remains

Very special symmetry, inherited from combination of:

- 5d Poincare invariance, and
- brane reparameterization invariance

$$\pi(x) \to \pi(x) + c + b_\mu x^\mu$$

The *Galilean* symmetry!

Beyond DGP

- Seems natural to consider extending to higher dimensions and to other models
- Several potential advantages of this
  - Might cure some of the ghost problems
  - Observations - stringent constraints on DGP model. In higher dimensions modifications to Friedmann equation should be milder - allow wider param range
  - Degravitation: gravity acts as a high-filter, suppressing contribution of vacuum energy to gravitational field. Too weak in DGP, more hopeful in D>5.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \longrightarrow G^{-1}(\Box) G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Galileons

Can consider this symmetry as interesting in its own right

- Yields a novel and fascinating 4d effective field theory
- Relevant field referred to as the *Galileon*

(Nicolis, Rattazzi, & Trincherini 2009)

$$\mathcal{L}_1 = \pi \qquad \mathcal{L}_2 = (\partial\pi)^2 \qquad \mathcal{L}_3 = (\partial\pi)^2 \Box\pi$$

$$\mathcal{L}_{n+1} = n\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}\left(\partial_{\mu_1}\pi\partial_{\nu_1}\pi\partial_{\mu_2}\partial_{\nu_2}\pi\cdots\partial_{\mu_n}\partial_{\nu_n}\pi\right)$$

There is a separation of scales

- Allows for classical field configurations with order one nonlinearities, but quantum effects under control.
- So can study non-linear classical solutions.
- Some of these are very important (Vainshtein effect)

Computing Feynman diagrams - terms of the galilean form cannot receive new contributions! More soon.

Luty, Porrati, Ratazzi (2003); Nicolis, Rattazzi (2004)

The Vainshtein Effect

Consider, for example, the DGP cubic term, coupled to matter

$$\mathcal{L} = -3(\partial\pi)^2 - \frac{1}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{M_{Pl}}\pi T$$

Now look at spherical solutions around a point mass

$$\pi(r) = \begin{cases} \sim \Lambda^3 R_V^{3/2}\sqrt{r} + const. & r \ll R_V \\ \sim \Lambda^3 R_V^3 \frac{1}{r} & r \gg R_V \end{cases} \qquad R_V \equiv \frac{1}{\Lambda}\left(\frac{M}{M_{Pl}}\right)^{1/3}$$

Looking at a test particle, strength of this force, compared to gravity, is then

$$\frac{F_\pi}{F_{\rm Newton}} = \frac{\pi'(r)/M_{Pl}}{M/(M_{Pl}^2 r^2)} = \begin{cases} \sim \left(\frac{r}{R_V}\right)^{3/2} & R \ll R_V \\ \sim 1 & R \gg R_V \end{cases}$$

So forces much smaller than gravitational strength within the Vainshtein radius - hence safe from 5th force tests.

The Vainshtein Effect

Suppose we want to know the the field that a source generates within the Vainshtein radius of some large body (like the sun, or earth)

Perturbing the field and the source

$$\pi = \pi_0 + \varphi, \quad T = T_0 + \delta T,$$

yields

$$\mathcal{L} = -3(\partial\varphi)^2 + \frac{2}{\Lambda^3}\underbrace{\left(\partial_\mu\partial_\nu\pi_0 - \eta_{\mu\nu}\Box\pi_0\right)}_{\sim \left(\frac{R_v}{r}\right)^{3/2}}\partial^\mu\varphi\partial^\nu\varphi - \frac{1}{\Lambda^3}(\partial\varphi)^2\Box\varphi + \frac{1}{M_4}\varphi\delta T$$

Thus, if we canonically normalize the kinetic term of the perturbations, we raise the effective strong coupling scale, and, more importantly, heavily suppress the coupling to matter!

Regimes of Validity

The usual quantum regime of a theory

$$r \ll \frac{1}{\Lambda}$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3/2} \gg 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \gg 1$$

$$\frac{1}{\Lambda} \ll r \ll R_V$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3/2} \gg 1$$

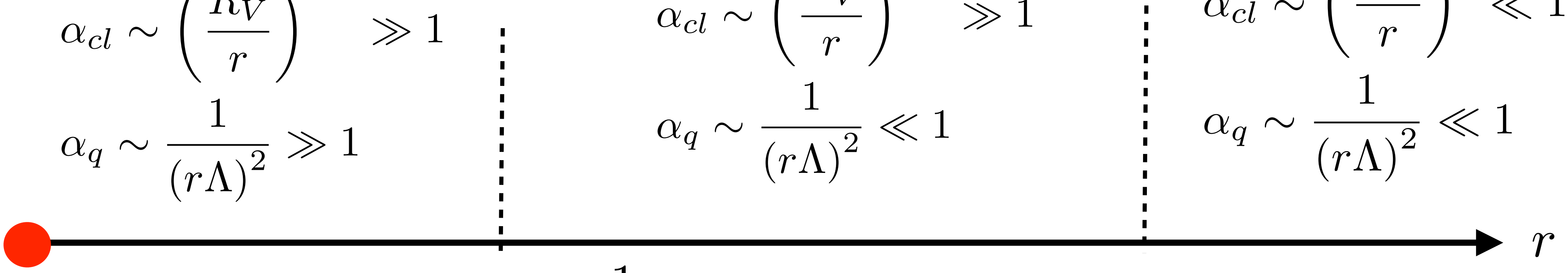
$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \ll 1$$

The usual linear, classical regime of a theory

$$r \gg R_V$$

$$\alpha_{cl} \sim \left(\frac{R_V}{r}\right)^{3} \ll 1$$

$$\alpha_q \sim \frac{1}{(r\Lambda)^2} \ll 1$$



$$r \sim \frac{1}{\Lambda}$$

$$r \sim R_V$$

A new classical regime, with order one nonlinearities

To be Specific ...

The Galilean terms take the form

$$\mathcal{L}_{n+1} = n\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n}\left(\partial_{\mu_1}\pi\partial_{\nu_1}\pi\partial_{\mu_2}\partial_{\nu_2}\pi\cdots\partial_{\mu_n}\partial_{\nu_n}\pi\right)$$

$$\eta^{\mu_1\nu_1\mu_2\nu_2\cdots\mu_n\nu_n} \equiv \frac{1}{n!}\sum_p (-1)^p\,\eta^{\mu_1 p(\nu_1)}\eta^{\mu_2 p(\nu_2)}\ldots\eta^{\mu_n p(\nu_n)}$$

- tensor is anti-symmetric in μ indices,
- anti-symmetric in ν indices, and
- symmetric under interchange of any μ, ν pair with any other

- Only first n of galileons terms non-trivial in n-dimensions.
- In addition, the tadpole term, π, is galilean invariant - include as the first-order galileon.

Interesting Mathematical Aside

The single field Galileon constitutes an example of what is known to mathematicians as an *Euler Hierarchy* [Thanks to David Fairlie]

Suppose have Lagrangian only depending on derivative:

$$L_1 = L_1(\dot{\phi}) \longrightarrow \mathcal{E}_1 = 0$$

$$L_2 = L_1\mathcal{E}_1 \longrightarrow \mathcal{E}_2 = 0$$

$$L_3 = L_2\mathcal{E}_2 \longrightarrow \mathcal{E}_3 = 0$$

$$\vdots$$

$$L_n = L_{n-1}\mathcal{E}_{n-1}$$

(total derivative)

Second order equations of motion, and series eventually terminates, as the Galileon one does

DBI Galileons and Conformal Galileons

Instead of extending Poincare symmetry by galilean one, might seek to extend to other useful symmetries. Making relativistic:

$$\delta\pi = c + b_\mu x^\mu - b^\mu \pi \partial_\mu \pi$$

DBI GALILEONS

makes full symmetry group P(4,1), spontaneously broken to P(3,1).
Again get n terms in n-dimensions, and the galileons in the small field limit

If we instead extend to the conformal group

$$\delta\pi = c - c x^\mu \partial_\mu \pi$$

$$\delta\pi = b_\mu x^\mu + \partial_\mu \pi \left(\frac{1}{2} b^\mu x^2 - (b \cdot x) x^\mu \right)$$

CONFORMAL GALILEONS

makes full symmetry group SO(4,2), spontaneously broken to P(3,1).
Again get n terms in n-dimensions. e.g.

$$\begin{aligned} \mathcal{L}_2 &= -\frac{1}{2} e^{-2\hat{\pi}} (\partial\hat{\pi})^2 \\ \mathcal{L}_3 &= \frac{1}{2} (\partial\hat{\pi})^2 \Box \hat{\pi} - \frac{1}{4} (\partial\hat{\pi})^4 \end{aligned}$$

Constructing Galileons: Probe Branes

[de Rham & Tolley]

Embed a flat 3-brane in a 5d flat bulk

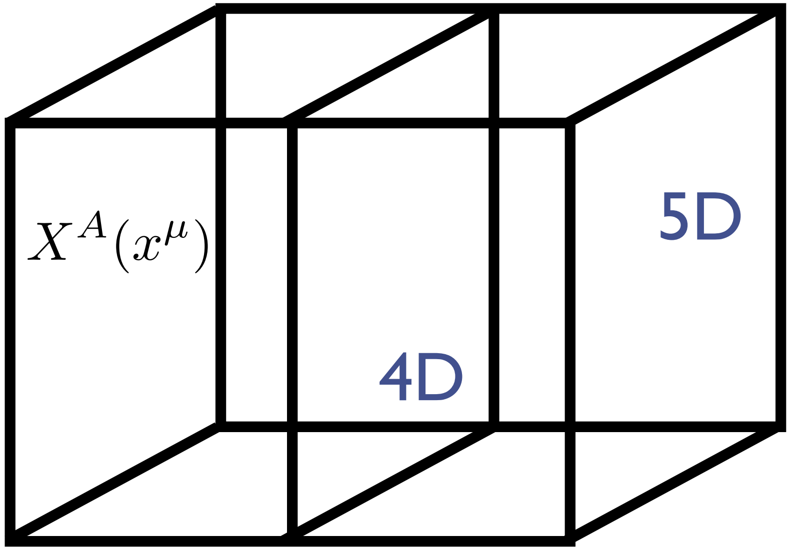
Symmetries are:

$$\delta_P X^A = \omega^A{}_B X^B + \epsilon^A$$

5d Poincare invariance

$$\delta_g X^A = \xi^\mu \partial_\mu X^A$$

Brane reparametrization invariance



Now pick a gauge

$$X^\mu(x) = x^\mu, \quad X^5(x) \equiv \pi(x)$$

A Poincare transformation ruins this choice, **but**: a simultaneous brane reparametrization restores it, so that the combination

$$\delta_{P'}\pi = \delta_P \pi + \delta_g \pi = -\omega^\mu{}_\nu x^\nu \partial_\mu \pi - \epsilon^\mu \partial_\mu \pi + \omega^5{}_\mu x^\mu - \omega^\mu{}_5 \pi \partial_\mu \pi + \epsilon^5$$

is still a symmetry

What remains is to construct actions

Actions in the Probe Brane Approach

The most general requirement is to us diffeomorphism invariant quantities on the brane.

$$S = \int d^4x \, \sqrt{-g} F\left(g_{\mu\nu}, \nabla_\mu, R^\rho{}_{\sigma\mu\nu}, K_{\mu\nu}\right)\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu\pi\partial_\nu\pi}$$

But we also want second order equations of motion. This restricts the form severely - to the Lovelock invariants and their associated Gibbons-Hawking-York boundary terms (Myers terms).

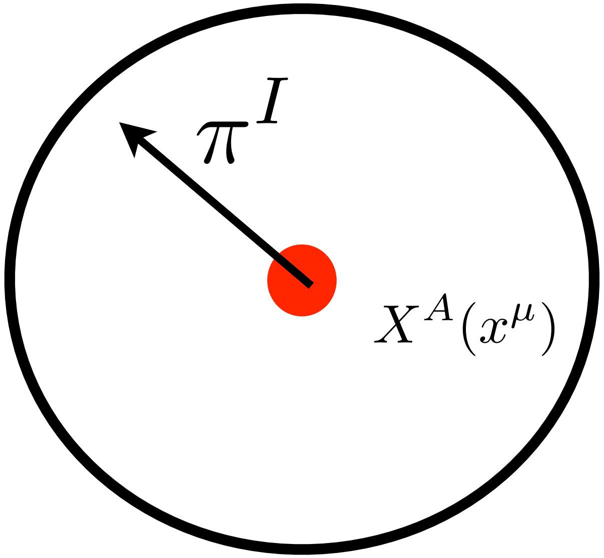
For example:

$$\int d^4x \, \sqrt{-g} \to \int d^4x \, \sqrt{1+(\partial\pi)^2}$$

This gives a DBI term, which in the small-field limit gives the second galileon term - the kinetic term.

Multi-field Galileons and Higher co-Dimension Branes

Higher co-Dimension Probe Branes



[K. Hinterbichler, M.T., D.Wesley, *Phys. Rev.* D82 (2010) 124018.]

With some work, can extend probe brane construction to multiple co-dimensions

$$X^\mu(x) = x^\mu, \quad X^I(x) \equiv \pi^I(x)$$

Induced Metric on Brane

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi^I \partial_\nu \pi_I$$

More general version of action de Rham & Tolley wrote

$$S = \int d^4x \sqrt{-g} F\left(g_{\mu\nu}, \nabla_\mu, R^i{}_{j\mu\nu}, R^\rho{}_{\sigma\mu\nu}, K^i_{\mu\nu}\right)\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu\pi^I\partial_\nu\pi_I}$$

Technical question. Main differences: extrinsic curvature $K^i_{\mu\nu}$ carries an extra index, associated with orthonormal basis in normal bundle to hypersurface.

Also, covariant derivative has connection, $\beta^j_{\mu i}$ acting on i index. e.g.

$$\nabla_\rho K^i_{\mu\nu} = \partial_\rho K^i_{\mu\nu} - \Gamma^\sigma_{\rho\mu} K^i_{\sigma\nu} - \Gamma^\sigma_{\rho\nu} K^i_{\mu\sigma} + \beta^i_{\rho j} K^j_{\mu\nu}$$

Higher co-Dimension Probe Branes

Covariant Derivative Intrinsic Curvature

$$S = \int d^4x \, \sqrt{-g} F\left(g_{\mu\nu}, \nabla_\mu, R^i{}_{j\mu\nu}, R^\rho{}_{\sigma\mu\nu}, K^i_{\mu\nu}\right)\Big|_{g_{\mu\nu}=\eta_{\mu\nu}+\partial_\mu\pi^I\partial_\nu\pi_I}$$

Normal Bundle Curvature

Extrinsic curvature

In co-dimension1, for 2nd order equations, use Lovelock terms and associated boundary terms. Here, for 4d brane, prescription *depends on co-dimension*

1. If N (not = 3) is odd, obtain dimensional continuation of Gibbons-Hawking and Myers terms, with the extrinsic curvature replaced by distinguished normal component of K.
2. If N = 3, have additional terms involving the extrinsic curvature (and boundary term is not simply dimensional continuation of Myers term.)
3. If N (not = 2) is even, boundary term includes only brane cosmological constant and induced Einstein-Hilbert term.
4. If N = 2, boundary terms include only brane cosmological constant, and

$$\mathcal{L}_{N=2} = \sqrt{-g}\left(R[g] - (K^i)^2 + K^i_{\mu\nu}K_i^{\mu\nu}\right)$$

The Multi-Galileon Limit

[K. Hinterbichler, M.T., D.Wesley, *Phys. Rev.* D82 (2010) 124018;
A.Padilla, P.Saffin, S.Zhou, JHEP 1012, 031 (2010).; C. Deffayet, S. Deser, G. Esposito-Farese, *Phys.Rev.* D82 (2010) 061501]

In decoupling limit get a unique multi-Galileon theory, with single coupling, from the brane Einstein-Hilbert action plus a brane cosmological constant:

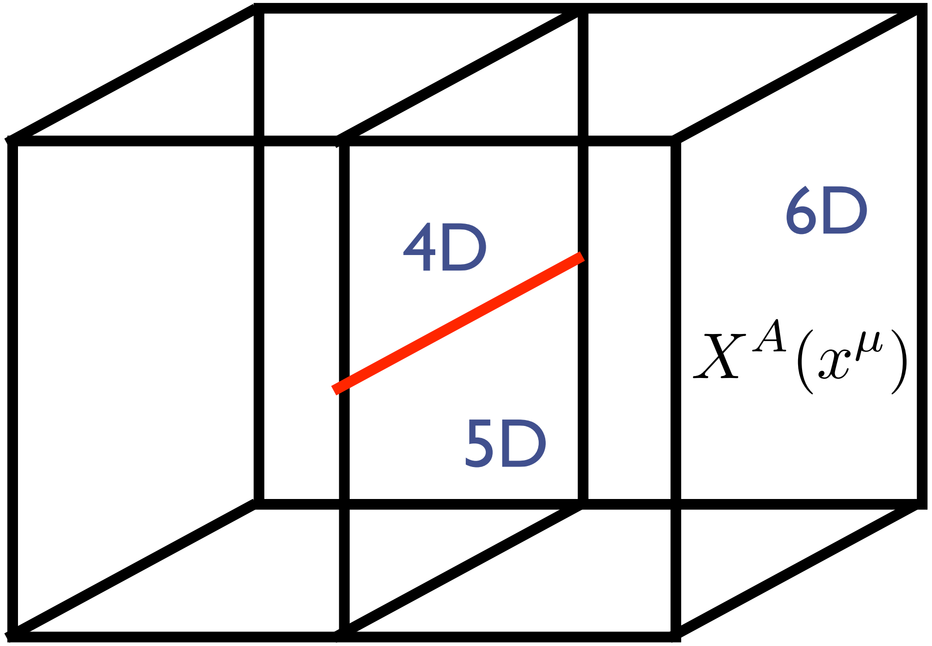
$$\int d^4x\,\sqrt{-g}\,(-a_2+a_4R)\rightarrow\int d^4x\,\left[-a_2\,\frac{1}{2}\partial_\mu\pi^I\partial^\mu\pi_I+a_4\;\partial_\mu\pi^I\partial_\nu\pi^J\left(\partial_\lambda\partial^\mu\pi_J\partial^\lambda\partial^\nu\pi_I-\partial^\mu\partial^\nu\pi_I\Box\pi_J\right)\right]$$

(In higher dimensions, more terms are possible)

As before, find combined symmetry in small-field limit under which π invariant:

$$\delta\pi^I=\underbrace{\omega^I{}_\mu x^\mu+\epsilon^I}_{\text{Multiple Galileons}}+\underbrace{\omega^I{}_J\pi^J}_{\text{New SO(N) symmetry}}$$

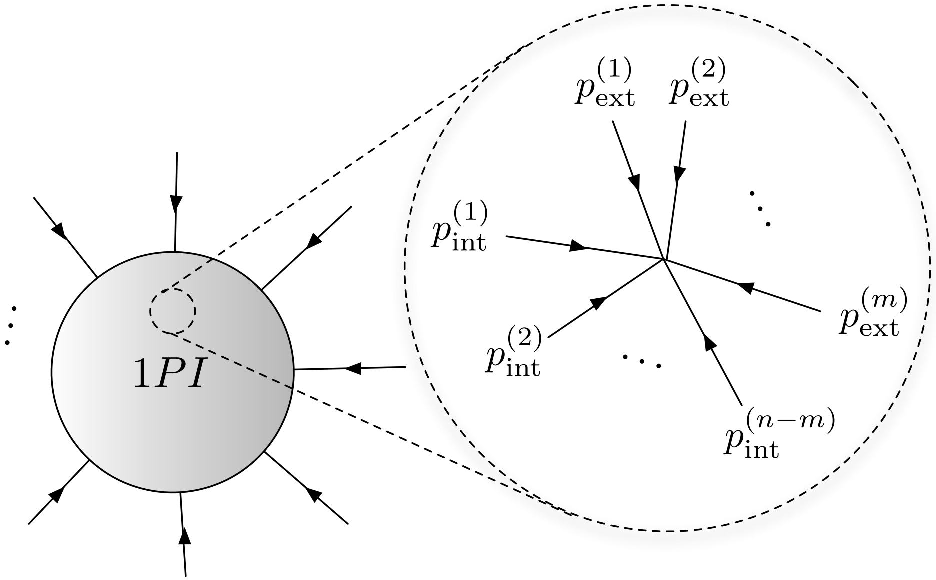
Breaking the SO(N) get a description more appropriate to, for example, cascading gravity.



Nonrenormalization!

Remarkable fact about these theories (c.f SUSY theories)

Expand quantum effective action for the classical field about expectation value



The n-point contribution contains at least 2n powers of external momenta: cannot renormalize Galilean term with only 2n-2 derivatives.

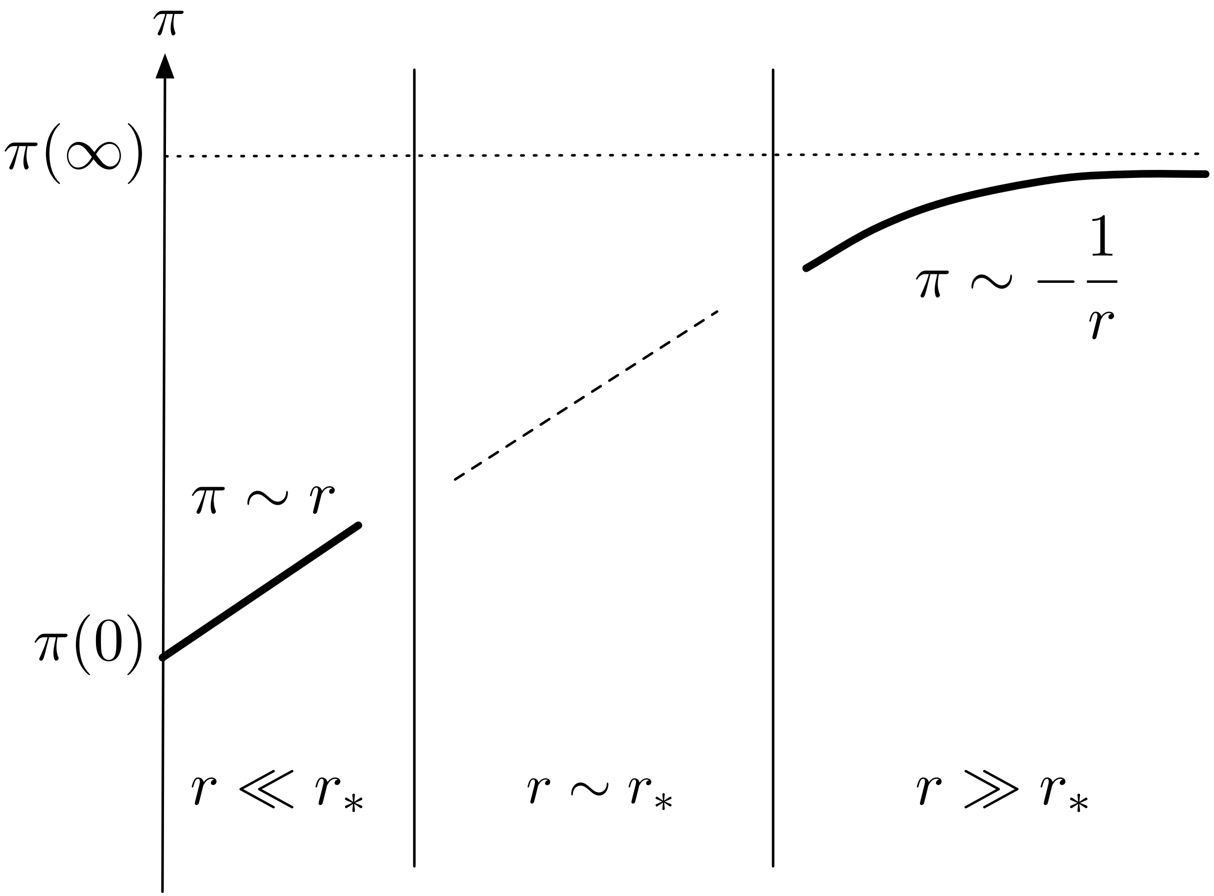
With or without the SO(N), can show, just by computing Feynman diagrams, that at all loops in perturbation theory, for any number of fields, terms of the galilean form cannot receive new contributions.

[K. Hinterbichler, M.T., D.Wesley, *Phys. Rev.* D82 (2010) 124018]

Can even add a mass term and remains technically natural

Coupling to Matter & Stability

[Andrews, Hinterbichler, Khoury, & M.T., *Phys.Rev.* D83 (2011) 044042]



For a single G

cts the symmetry

For multi-Gal

$\pi^I \pi^I T$

isn't invariant.

Simplest invar

has no nontri

sources

But for exam

interaction

$\mathcal{L} =$

Solve

$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^3\left(y^I\right.\right.$

$y^I \equiv \frac{1}{r}\frac{\mathrm{d}\pi^I}{\mathrm{d}r}$

Solution has a Vainshtein radius $\left(\lambda M^2\left[P'\left(\pi^2(0)\right)\pi(0)\right]^2\right)^{1/6}$

BUT: exhibits superluminality and instability. If these are to make sense, better couplings to matter are needed.

Generalized Galileons on Curved Geometries: Cosmological Spaces

Galileons on General Backgrounds

[Goon, Hinterbichler, M.T., *Phys. Rev.Lett.* 106, 231102 (2011).
Goon, Hinterbichler, M.T., *JCAP* 1107, 017 (2011).]

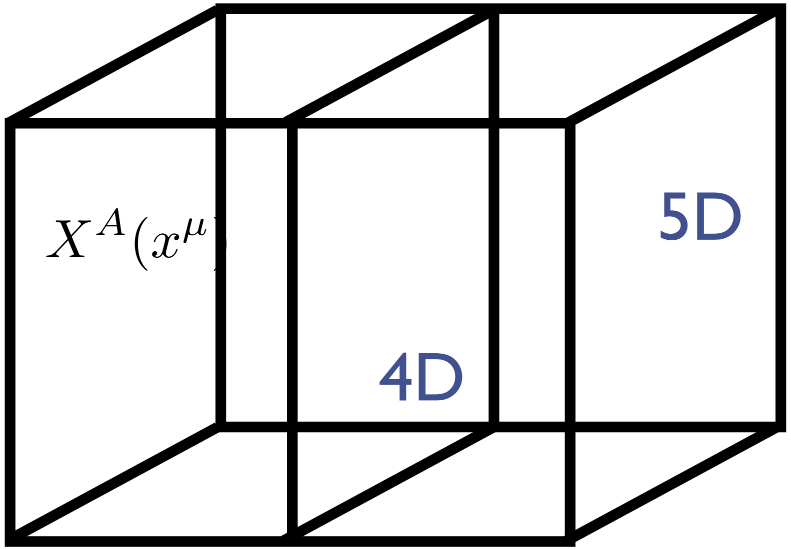
Main point:

- Can extend probe brane construction to more general geometries. e.g. other maximally-symmetric examples

Bulk $$ds^2 = d\rho^2 + f(\rho)^2 g_{\mu\nu}(x) dx^\mu dx^\nu$$

Induced on Brane $$\bar{g}_{\mu\nu} = f(\pi)^2 g_{\mu\nu} + \nabla_\mu \pi \nabla_\nu \pi$$

Bulk Killing Vectors $$\delta_K X^A = a^i K_i^A(X) + a^I K_I^A(X)$$



Galileons with symmetry

$$(\delta_K + \delta_{g,\text{comp}})\pi = -a^i k_i^\mu(x)\partial_\mu \pi + a^I K_I^5(x,\pi) - a^I K_I^\mu(x,\pi)\partial_\mu \pi$$

The Maximally-Symmetric Taxonomy

Potentially different Galileons corresponding to different ways to foliate a maximally symmetric 5-space by a maximally symmetric 4-d hypersurface

Brane metric

Ambient metric	AdS_4	M_4	dS_4
AdS_5	AdS DBI galileons $so(4,2) \rightarrow so(3,2)$ $f(\pi) = \mathcal{R}\cosh^2(\rho/\mathcal{R})$	Conformal DBI galileons $so(4,2) \rightarrow p(3,1)$ $f(\pi) = e^{-\pi/\mathcal{R}}$	type III dS DBI galileons $so(4,2) \rightarrow so(4,1)$ $f(\pi) = \mathcal{R}\sinh^2(\rho/\mathcal{R})$
M_5	×	DBI galileons $p(4,1) \rightarrow p(3,1)$ $f(\pi) = 1$	type II dS DBI galileons $p(4,1) \rightarrow so(4,1)$ $f(\pi) = \pi$
dS_5	×	×	type I dS DBI galileons $so(5,1) \rightarrow so(4,1)$ $f(\pi) = \mathcal{R}\sin^2(\rho/\mathcal{R})$
Small field limit ↓	AdS galileons	normal galileons	dS galileons

Galileons on Gaussian Normal Foliations

[Goon, Hinterbichler, M.T., JCAP 1112 (2011) 004 [1109.3450 [hep-th]]]

Can we foliate a 5-d space in an interesting way such that the resulting theory describes galileons with the appropriate symmetries to propagate on a Friedmann, Robertson-Walker (FRW) background?

- Can actually do a little better - can do a general Gaussian Normal foliation

$$G_{AB}dX^A dX^B = f_{\mu\nu}(x,w)dx^\mu dx^\nu + dw^2$$

$$\bar{g}_{\mu\nu} = f_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi$$

Induced
on Brane

$$\begin{aligned}
\mathcal{L}_1 &= \int^{\pi(x)} d\pi' \sqrt{-\det f_{\mu\nu}(x,\pi')}, \\
\mathcal{L}_2 &= -\sqrt{-f}\frac{1}{\gamma}, \\
\mathcal{L}_3 &= \sqrt{-f}\left[-\langle\Pi\rangle + \frac{1}{2}\langle f'\rangle + \gamma^2\left(\langle\pi\Pi\pi\rangle + \frac{1}{2}\langle\pi f'\pi\rangle\right)\right], \\
\mathcal{L}_4 &= \sqrt{-f}\Big[-\frac{1}{2}\langle\pi f'\pi\rangle^2\gamma^3 - \langle f'\rangle\langle\pi\Pi\pi\rangle\gamma^3 - 2\langle\pi\Pi^2\pi\rangle\gamma^3 + 2\langle\pi\Pi\pi\rangle\langle\Pi\rangle\gamma^3 \\
&\quad - \frac{1}{2}\langle f'\rangle\langle\pi f'\pi\rangle\gamma^3 + \langle\Pi\rangle\langle\pi f'\pi\rangle\gamma^3 - \frac{\langle f'\rangle^2\gamma}{4} - \langle\Pi\rangle^2\gamma + \frac{\langle f'^2\rangle\gamma}{4} \\
&\quad - \langle\Pi f'\rangle\gamma + \langle f'\rangle\langle\Pi\rangle\gamma + \langle\Pi^2\rangle\gamma + \frac{\langle\pi f'^2\pi\rangle\gamma}{2}\Big],
\end{aligned}$$

Embedding 4d FRW in 5d Minkowski

$$ds^2 = -\left(dY^0\right)^2 + \left(dY^1\right)^2 + \left(dY^2\right)^2 + \left(dY^3\right)^2 + \left(dY^5\right)^2$$

$$Y^0 = S(t,w)\left(\frac{x^2}{4} + 1 - \frac{1}{4H^2a^2}\right) - \frac{1}{2}\int dt\, \frac{\dot{H}}{H^3 a},$$

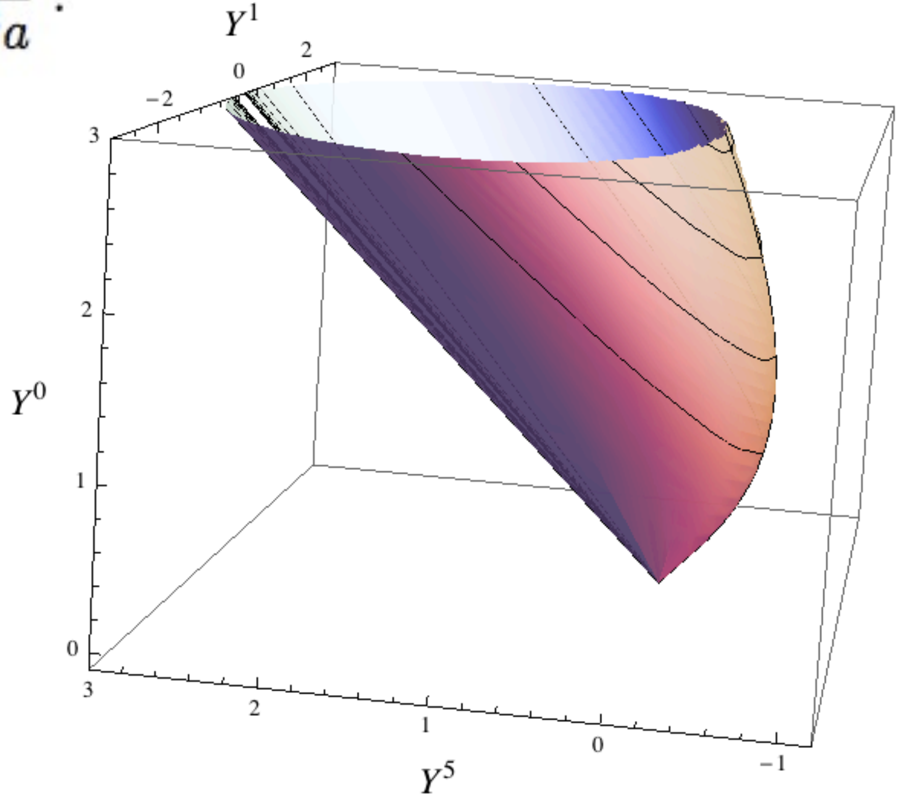
$$Y^i = S(t,w)x^i,$$

$$Y^5 = S(t,w)\left(\frac{x^2}{4} - 1 - \frac{1}{4H^2a^2}\right) - \frac{1}{2}\int dt\, \frac{\dot{H}}{H^3 a}\ .$$

$$S(t,w) \equiv a - \dot{a}w$$

Induced Metric on Brane

$$d\tilde{s}^2 = -n^2(t,w)dt^2 + S^2(t,w)\delta_{ij}dx^i dx^j$$



Galileons on Cosmological Backgrounds

[Goon, Hinterbichler, M.T., JCAP 1112 (2011) 004 [1109.3450 [hep-th]]]

The form of the first two Lagrangians, for example, is

$$\mathcal{L}_1 = a^3\pi - \frac{a^2\left(3\dot{a}^2 + a\ddot{a}\right)\pi^2}{2\dot{a}} + a\left(\dot{a}^2 + a\ddot{a}\right)\pi^3 - \frac{1}{4}\dot{a}\left(\dot{a}^2 + 3a\ddot{a}\right)\pi^4 + \frac{1}{5}\ddot{a}\dot{a}^2\pi^5,$$

$$\mathcal{L}_2 = -(1 - \frac{\ddot{a}}{\dot{a}}\pi)(a - \dot{a}\pi)^3\sqrt{1 - \left(1 - \frac{\ddot{a}}{\dot{a}}\pi\right)^{-2}\dot{\pi}^2 + (a - \dot{a}\pi)^{-2}(\vec{\nabla}\pi)^2}.$$

and the symmetries are

$$\begin{aligned}
\delta_{v_i}\pi &= \frac{1}{2}x^i\dot{a}\int dt\,\frac{\dot{H}}{H^3 a} - \frac{x^i\left(a - \dot{a}\pi + \dot{a}^2\int dt\,\frac{\dot{H}}{H^3 a}\right)}{2\dot{a} - 2\pi\ddot{a}}\dot{\pi} \\
&\quad + \left[\frac{x^i x^i \dot{a}^2 + 1}{4\dot{a}^2} + \frac{\int dt\,\frac{\dot{H}}{H^3 a}}{2a - 2\pi\dot{a}}\right]\partial_i\pi - \sum_{j\neq i}\left[-\frac{x^i x^j}{2}\partial_j\pi + \frac{x^j x^j}{4}\partial_i\pi\right] \\
\delta_{k_i}\pi &= x^i\dot{a}\left(\frac{\dot{a}\dot{\pi}}{\dot{a} - \pi\ddot{a}} - 1\right) - \frac{\partial_i\pi}{a - \pi\dot{a}}, \\
\delta_q\pi &= \frac{\dot{\pi}\dot{a}^2}{\pi\ddot{a} - \dot{a}} + \dot{a}, \\
\delta_u\pi &= \frac{x^2\dot{a}^2 - 1}{4\dot{a}} - \frac{x^2\dot{a}^2 + 1}{4\dot{a} - 4\pi\ddot{a}}\dot{\pi} + \frac{1}{2a - 2\pi\dot{a}}\sum_i x^i\partial_i\pi, \\
\delta_s\pi &= -\dot{a}\int dt\,\frac{\dot{H}}{H^3 a} + \frac{\left(a - \dot{a}\pi + \dot{a}^2\int dt\,\frac{\dot{H}}{H^3 a}\right)\dot{\pi}}{\dot{a} - \pi\ddot{a}} - \sum x^i\partial_i\pi,
\end{aligned}$$

These describe covariant versions of Galileons, naturally propagating on FRW backgrounds.

Simple Solutions and Stability

Expand Lagrangians to second order in π, and integrate by parts (a lot)

$$\mathcal{L}_1 = a^3\pi - \frac{1}{2}\left(\frac{\ddot{a}a^3}{\dot{a}} + 3\dot{a}a^2\right)\pi^2 + \mathcal{O}(\pi^3)$$

$$\mathcal{L}_2 = (3a^2\dot{a} + \frac{a^3\ddot{a}}{\dot{a}})\pi + \frac{1}{2}a^3\dot{\pi}^2 - \frac{1}{2}a(\vec{\nabla}\pi)^2 - 3\left(\ddot{a}a^2 + \dot{a}^2a\right)\pi^2 + \mathcal{O}(\pi^3)$$

$$\mathcal{L}_3 = 6(a\dot{a}^2 + a^2\ddot{a})\pi + 3\dot{a}a^2\dot{\pi}^2 - \left(2\dot{a} + \frac{a\ddot{a}}{\dot{a}}\right)(\vec{\nabla}\pi)^2 - 3\left(3\dot{a}\ddot{a}a + \dot{a}^3\right)\pi^2 + \mathcal{O}(\pi^3)$$

$$\mathcal{L}_4 = 6(\dot{a}^3 + 3a\dot{a}\ddot{a})\pi + 9\dot{a}^2a\dot{\pi}^2 - 3\left(\frac{\dot{a}^2}{a} + 2\ddot{a}\right)(\vec{\nabla}\pi)^2 - 12\dot{a}^2\ddot{a}\pi^2 + \mathcal{O}(\pi^3)$$

$$\mathcal{L}_5 = 24\dot{a}^2\ddot{a}\,\pi + 12\dot{a}^3\dot{\pi}^2 - 12\frac{\ddot{a}^2\dot{a}}{a}(\vec{\nabla}\pi)^2 + \mathcal{O}(\pi^3)$$

Write

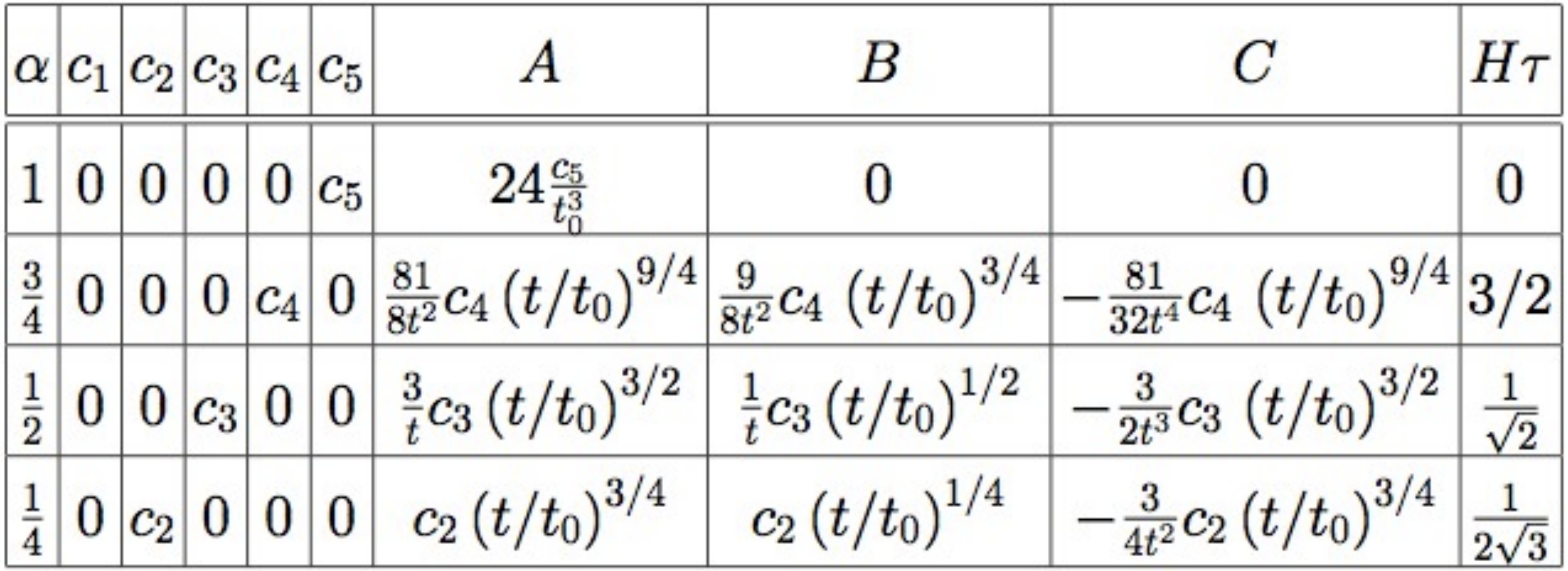
$$\mathcal{L} = \sum_{n=1}^{5} c_n\mathcal{L}_n$$

and just for example, look for combinations for which π=0 is a solution

Fix $a(t) = (t/t_0)^{\alpha}$ π=0 solutions exist for $\alpha = 1, 3/4, 1/2, 1/4$

Expanding to quadratic order about solution yields
(note - no higher derivatives - one degree of freedom!)

$$\mathcal{L} = \frac{1}{2}A(a(t), c_n)\dot{\pi}^2 - \frac{1}{2}B(a(t), c_n)(\vec{\nabla}\pi)^2 - \frac{1}{2}C(a(t), c_n)\pi^2$$



α	c_1	c_2	c_3	c_4	c_5	A	B	C	$H\tau$
1	0	0	0	0	c_5	$24\frac{c_5}{t_0^3}$	0	0	0
$\frac{3}{4}$	0	0	0	c_4	0	$\frac{81}{8t^2}c_4\,(t/t_0)^{9/4}$	$\frac{9}{8t^2}c_4\,(t/t_0)^{3/4}$	$-\frac{81}{32t^4}c_4\,(t/t_0)^{9/4}$	3/2
$\frac{1}{2}$	0	0	c_3	0	0	$\frac{3}{t}c_3\,(t/t_0)^{3/2}$	$\frac{1}{t}c_3\,(t/t_0)^{1/2}$	$-\frac{3}{2t^3}c_3\,(t/t_0)^{3/2}$	$\frac{1}{\sqrt{2}}$
$\frac{1}{4}$	0	c_2	0	0	0	$c_2\,(t/t_0)^{3/4}$	$c_2\,(t/t_0)^{1/4}$	$-\frac{3}{4t^2}c_2\,(t/t_0)^{3/4}$	$\frac{1}{2\sqrt{3}}$

Either marginally stable, or a tachyonic instability, with tachyon timescale ~1/H. Therefore, solutions stable to fluctuations over time scales shorter than the age of the universe.

[Agrees with Burrage, de Rham, and Heisenberg, JCAP 1105 (2011) 025, arXiv:1104.0155.]

Galileon-Like Limit

In maximally symmetric case have small field limits which simplify Lagrangians (To obtain, form linear combinations of original Lagrangians, s.t. perturbative expansion of nth one around constant background order π^n) e.g. flat brane in a flat bulk gives flat space galileons.

Can't do same here - appears to be due to maximal symmetry, but can check our results for dS limit:

Induced Metric on Brane $\bar{g}_{\mu\nu} = (-1 + H\pi)^2 g^{(dS)}_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi$

Now redefine the field and change coordinates

$$\tilde{\pi} = -1 + H\pi \qquad \hat{x}^\mu = H x^\mu$$

Resulting theory is one of the ones I mentioned earlier, and the small field limit is the resulting Galileon on a dS background - reassuring!

Applications

•At this point there are a reasonably large number of fledgling attempts to apply these ideas to cosmology, field theory, and gravity

- Early cosmology and inflation. Galileon inflation - radiatively stable - operators protected by covariant version of Galileon symmetry. Potential test via nongaussianity (e.g. Burrage, de Rham, Seery and Tolley 2010)
- Galilean genesis (alternative to inflation); and in general as a way to violate the null energy condition.
- A possible well-behaved way to modify gravity, perhaps in the infrared (degravitation?). See also Fab Four
- Supersymmetrization
- An appearance in the decoupling limit of some massive gravity theories

Summary

- Higher dimensional models are teaching us about entirely novel 4d effective field theories that may be relevant to cosmology
- We have shown how to derive the scalar field theories corresponding to Galileons propagating on fixed curved backgrounds (maximally symmetric and FRW examples).
- Have also shown how to extend the probe brane construction to higher co-dimension branes, yielding multi-Galileon theories.
- Couplings to matter and stability still need investigating in generality.

Current Work & the Future

- Galileons are Wess-Zumino terms! In d dimensions are d-form potentials for (d+1)-forms which are non-trivial co-cycles in Lie algebra cohomology of full symmetry group relative to unbroken one. Slightly different stories for DBI and conformal Galileons. [Goon, Hinterbichler, Joyce & M.T., arxiv:1203.3191 [hep-th])

- Our models tell you what Galileons do propagating on cosmological spaces. What about driving cosmology? Need dynamical gravity for that, and we think we know how to do this (ongoing work w/ Hinterbichler, Khoury & Gabadadze).

- What lies behind the nonrenormalized Lagrangians?

Thank You!

- Many of the questions I raised regarding cosmology.